

Dynamic Neural Fields with Intrinsic Plasticity

Claudius Strub^{1,2*}, Gregor Schöner¹, Florentin Wörgötter² and Yulia Sandamirskaya³

¹Autonomous Robotics Lab, Institut für Neuroinformatik, Ruhr-Universität, Bochum, Germany

 ²Department of Computational Neuroscience, III Physics Institute, Georg-August-Universität, Göttingen, Germany
 ³Institute of Neuroinformatics, University of Zurich, Zurich, Switzerland

Correspondence*: Claudius Strub Institut für Neuroinformatik, Ruhr-Universität Bochum, Universitätsstr. 150, 44780 Bochum, Germany, claudius.strub@ini.rub.de

2 ABSTRACT

1

Dynamic neural fields (DNFs) are dynamical systems models that approximate the activity of 3 4 large, homogeneous, and recurrently connected neural networks based on a mean field approach. Within dynamic field theory, the DNFs have been used as building blocks in architectures to 5 model sensorimotor embedding of cognitive processes. Typically, the parameters of a DNF in an 6 7 architecture are manually tuned in order to achieve a specific dynamic behavior (e.g. decision making, selection, or working memory) for a given input pattern. This manual parameter search 8 requires expert knowledge and time to find and verify a suited set of parameters. The DNF 9 parametrization may be particular challenging if the input distribution is not known in advance, 10 e.g. when processing sensory information. In this paper, we propose an autonomous adaptation 11 of the DNF resting level and gain by a learning mechanism of intrinsic plasticity (IP). To enable 12 13 this adaptation, an input and output measure for the DNF are introduced, together with a hyper parameter to define the desired output distribution. The online adaptation by IP gives the possibility 14 15 to pre-define the DNF output statistics without knowledge of the input distribution and thus, also to compensate for changes in it. The capabilities and limitations of this approach are evaluated 16 in a number of experiments, including a comparison of stochastic gradient with natural gradient 17 18 descent.

19 Keywords: Dynamic Neural Fields, Intrinsic Plasticity, Adaptation, Dynamics

1 INTRODUCTION

A Dynamic Neural Field is a description of activity of a large homogeneous neuronal population (Amari, 1977; Wilson and Cowan, 1973; Coombes et al., 2014; Schöner and Spencer, 2015). The DNF equation is obtained as a mean-field approximation of the dynamics of a network of spiking neurons and describes the dynamics of a continuous *activation* function, spanned over a *feature dimension*, such as color, location, velocity, or other perceptual or motor parameters, to which the neurons in the underlying population respond.

The core elements of the DNF dynamics are a winner-takes-all type of connectivity, expressed by a 26 27 symmetrical interaction kernel with a short-range excitation and a long-range inhibition (a "Mexican Hat" connectivity pattern), and a sigmoidal non-linearity. The sigmoidal non-linearity determines the *output* of 28 the DNF. The DNF's output is a function over the feature dimension that vanishes for activation values 29 below zero and saturates at one for positive activation values. The recurrent connectivity pattern and 30 sigmoid output function of the DNF lead to non-linear properties of this model. These properties enabled 31 its successful application in modeling cognitive functions in humans: e.g., formation of a representation, 32 working memory, decision making, rule learning, or executive control (Schöner and Spencer, 2015), as 33 34 well as for control of cognitive robots (Sandamirskaya et al., 2013; Bicho et al., 2011).

35 One of the obstacles to a wider adoption of the DNF model in technical systems and in neurobehavioral modelling is the parameter tuning required to obtain the desired behavior. In general, the behavior of a DNF 36 for a given input depends on the parameters of the neural field, e.g. the strength and width of the interaction 37 kernel, the resting level, or the slope of the sigmoidal non-linearity. However, when considering the DNF 38 39 output behavior over time for sequences of inputs, the particular input distribution has a major impact 40 on the DNF output statistics. Therefore, the input distribution has to be taken into account when setting 41 parameters of a DNF in order to achieve a desired behavior over time. When the input distribution is not 42 known in advance and its online normalization is not straight-forward, the tuning of the DNF parameters may be time consuming, in particular when the input distribution varies (e.g. drifts) over time. 43

44 Let us consider an example that we will use throughout this paper: a robotic hand with a tactile sensor on its fingertip (Shunk Dexterous Hand 2, SDH-2) is used to estimate the shape of an object by rotating 45 46 it and bringing the fingertip in contact with the object in different locations. In this example, we use the tactile sensor on the robotic fingertip as a source of input to a DNF (see Strub et al. (2014b,a) for the 47 details of the robotic setup and the DNF architecture). When the tactile sensor is brought into contact with 48 the object, the shape of the contact area is characteristic for the properties of the object's surface at the 49 location of the contact. A low circularity of the contact area on a tactile sensor (down to zero circularity for 50 a sharp line) corresponds to an edge on the object's surface, whereas high circularity (up to 1 for a perfect 51 circle) corresponds to a flat surface under the sensor. The task we will consider is to create a "map" of 52 flat surfaces of an object (in an object-centered coordinate frame), as the robotic hand rotates the object, 53 repeatedly bringing the tactile sensors on its fingertips in contact with the object at different locations. To 54 build such "map", we need to detect the most circular contacts within a sequence of contacts and store their 55 positions on the object's map. Here, the angular coordinate anchored in the object's center is the feature 56 dimension we are interested in (on which we build a "map" of the detected flat surfaces, using the DNF). 57 The circularity of the contact point determines the activation level of the DNF, induced by the sensory 58 input. The desired output of the DNF in this case is an activity peak for a given fraction of the inputs (e.g., 59 within 20% of the most circular inputs), and no output activity for inputs with lower circularity values. 60

The parametrization of such a detector depends crucially on the distribution of the circularity feature in 61 the input stream from the sensor, as illustrated in Fig. 1. In the figure, the sensed property – circularity in 62 our example, - which determines activation level of the DNF, is plotted on the x-axis and its probability 63 of occurrence on the y-axis, thus showing the distribution of input amplitudes that can be measured. 64 The green-colored part of the distribution corresponds to a fixed fraction of the input distribution, say 65 20% that includes the highest input circularities. The three examples shown in Fig. 1, illustrate that the 66 classification threshold for the activation of a DNF – which is determined by the negative resting level 67 and the threshold parameter of the sigmoid of the DNF – depends on the particular feature distribution. If 68 the input distribution is not known in advance, or varies over time, an online adaptation of the detection 69



Figure 1. Illustration of three different distributions of a "circularity feature" obtained from sensory input. On the horizontal axis, the circularity is denoted, which determines the activation level of a DNF; the vertical axis shows the probability to measure the respective circularity value. The green filling represents a fixed fraction (20%) of the total probability density.

threshold (i.e., resting level) and steepness of the classification function (i.e., steepness of the sigmoid) is
necessary.

72 In this paper, we propose a method to autonomously adapt parameters of a DNF - in particular, the gain of the sigmoid non-linearity and the resting level (bias) – using a homeostatic process. In order to achieve 73 this adaptation, global input and output measures for the DNF have to be defined. Here, we use as an output 74 75 measure the maximum level of the output of the field (output is the activation after it is passed through the sigmoid function). The corresponding input measure is the activation value of the input at the location 76 of the maximum output. The underlying notion is that the maximum level of the DNF output reflects 77 the decision that the field has made about its input. That decision was based on the input at the selected 78 location. Based on these measures, the gain and resting level for the DNF are adapted in order to match the 79 distribution of this output measure obtained over time to a predefined target distribution. This adaptation 80 drives the DNF dynamics towards the detection instability, which separates the inactive, subthreshold states 81 of the DNF from the active states with a local activation peak. As a result, the DNF is kept in a dynamical 82 regime in which it remains sensitive to input, preventing both saturation and the complete absence of 83 activity. Furthermore, the adaptation ensures that the distribution of the output measure of the DNF remains 84 invariant when the input distribution changes over time, for example, in terms of its mean or variance. 85

In the following sections, the DNF and IP equations are introduced, the derivation of DNFs with IP is outlined, and the performance of the modified DNF is evaluated on an example, in which input from a tactile sensor is processed.

2 METHODS

89 2.1 Dynamic Neural Fields

90 DNFs are dynamical systems which model activation dynamics in large homogeneously connected recurrent neuronal networks. The DNF equation describes an activation function that may represent a 91 92 perceptual feature, location in space, or a motor control variable (Schöner and Spencer, 2015). This 93 behavioral variable is encoded along a feature dimension x of the DNF, and the activation u(x, t) at position x encodes the confidence that the feature has value x at time t. The current state of the neuronal system is 94 encoded by the position on the dimension x of high activation values. Such space coding allows to encode 95 multiple possible values of a feature as well as "fuzziness" of the experienced or stored values. Other 96 artificial neural networks encode feature values through the level of activity of particular neuronal units 97 (rate coding) or through the pattern of activation across a distributed set of units. What is special about the 98 space code used in DNF is that the metric distance between represented values is explicitly encoded in 99 the distance between locations along the feature dimension. Neural coupling that depends on the distance 100 between field locations thus depends on the distance in feature space between the represented feature 101 102 values.

103 The equation for the DNFs used in the proposed model is shown in Eq. (1) and defines the rate of change 104 of the activation u(x, t) of the field:

$$\tau \dot{u}(x,t) = -u(x,t) + h + S(x,t) + \int \omega(|x-x'|)g(u(x',t))dx'.$$
(1)

In Eq. (1), u(x, t) is the activation of the DNF at time step t and position x. The positions x belong to a feature dimension, and can also span a multi-dimensional feature space: $\vec{x} \in \mathbb{R}^n$ (Schöner and Spencer, 2015). In practice, the dimensionality of fields ranges from zero (neural nodes) to three or four. In this paper only one dimensional fields are considered.

109 The term -u(x,t) stabilizes an attractor for the activation function at values that are defined by the last 110 three terms in the equation. The time constant, τ , determines how fast activation patterns, u(x,t), relaxes 111 to the attractor. The negative resting level, h, ensures that the DNF produces no output in the absence of 112 external input, S(x,t). The convolution term models recurrent neural interactions between activation levels 113 at different locations within the DNF, and is shaped by the interaction kernel:

$$\omega(|x - x'|) = c_{exc} \exp\left[-\frac{(x - x')^2}{2\sigma_{exc}^2}\right] - c_{inh} \exp\left[-\frac{(x - x')^2}{2\sigma_{inh}^2}\right],$$
(2)

114 with a short-range excitation (strength c_{exc} , width σ_{exc}) and a long-range inhibition (strength c_{inh} , width 115 $\sigma_{inh} > \sigma_{exc}$). A sigmoidal non-linearity, $g(u(x,t)) = (1 + \exp[-\beta u(x,t)])^{-1}$ defines the output of the 116 DNF through which the DNF impacts on other neural dynamics within a neural architecture, and also on 117 its own neural dynamics through the recurrent interactions.

118 The -u(x,t) in Eq. (1) guarantees the existence of at least one attractor. Dependent on the parametrization 119 of the recurrent interaction kernel ω and the sum of the input signal S and the resting level h, the DNF 120 may undergo saddle-node bifurcations. Fig. 2 shows schematically the bifurcations that a DNF undergoes 121 when the sum h + S(x) changes. The left column of the figure shows a zero-dimensional case (when u is a 122 scalar value and the state is a point), and the right column shows a one-dimensional case (when u(x) is a 123 function and the state corresponds to a line).

The phase plots on the left in Fig. 2 qualitatively show Eq. (1) for a zero-dimensional state x (i.e. a point) at different input values:

- In the top plot ("Low-stable"), the black dot denotes a single stable fixed point (attractor), marked with
 a, for the case when h + S is below activation threshold of the DNF. Loosely speaking, the resting
 level h together with the input S of the DNF shift the function of the phase plot up and down, while ω
 determines the non-linearity of the function whose general shape is determined by the sigmoid g.
- A stronger input intensity S may cause a bifurcation, creating two new fixed points: a stable one (point c) and an unstable one (point b) ("Bi-stable" regime, middle plot).
- If the input further increases, a second bifurcation occurs, where the unstable fixed point (b) collides with the stable fixed point (a) in the phase plot. Now the system state at the former fixed point (a) has lost its stability and the system will converge to the remaining stable fixed point (c) ("High-stable", lower plot). This second bifurcation, where the bi-stable state looses stability and switches to the high-stable state, is termed *detection instability*.



Figure 2. The three regimes of stability. Left column: phase plots for different regimes of the DNF equation for a zero-dimensional feature space x (u is a scalar value). Black dots indicate stable fixed points, empty circles – unstable fixed points. Right column: the output g(u) of a DNF is illustrated (in red) for an one-dimensional feature space x. The blue-dashed line represents the input S(x). The arrows depict qualitative changes in the regimes of stability determined by the input strength S(x).

If the input S now decreases again, it will induce a bifurcation, leading back to the bi-stable regime;
however, the system will remain at the stable fixed point (c) (hysteresis). Only if the input decreases
enough to induce the second bifurcation, where the stable fixed point (c) collides with the unstable
fixed point (b), the system will return to the stable fixed point (a) ("Low-stable").

141 In the one-dimensional system in the right column of Fig. 2, the output activation q(x) over the feature space x is plotted in red while the input S(x) is plotted as a dashed blue line. In the "Low-stable" case, 142 when the sum h + S(x) does not reach activation threshold anywhere in the dimension x, the output of the 143 DNF is zero (top plot A1). If the input intensity increases, the system enters a "Bi-stable" regime (middle 144 plot A2), with a weak positive output. When input strength further increases, the detection instability leads 145 to a visible change of the DNF output, which has now a localised "peak" that might even surpass the 146 input strength (lower plot, C1). If the input intensity is decreased now, the system will enter the bi-stable 147 regime again (middle plot, C2), however, without a qualitative change in the system output. The system 148 output drops back to the resting level activity in (A1) only if input is further decreased, which is termed 149 the "reverse detection instability". For high values of lateral excitation, a negative (inhibitory) input is 150 required for the system to return to an inactive state. This parametrization is termed "self-stabilizing", i.e. 151 152 maintaining the output in the absence of input S(x, t) = 0.

To conclude, the recurrent interactions shaped by the kernel ω stabilize the system in its state (either 154 "low" or "high") when the input fluctuates around the bistable setting, by shaping the non-linearity in the 155 phase-characteristics of the system's dynamics.

156 2.2 Intrinsic Plasticity

Neurons in biological organisms have a large spectrum of plasticity mechanisms, implementing a 157 broad range of functions. One functional class of neuronal plasticity mechanisms is termed "homeostatic 158 plasticity", which optimizes the information processing within a neuron by keeping the firing rate of the 159 neuron in a reasonable regime Turrigiano (2008); Pozo and Goda (2010); Turrigiano (2012). Non-synaptic, 160 i.e. intrinsic forms of homeostatic plasticity are termed "intrinsic homeostatic plasticity" (IP), which 161 162 adapt the intrinsic excitability of a neuron (Frick and Johnston, 2005; Schulz, 2006). Additionally to the adaptation in the neuron soma, this plasticity of excitability has also been discovered in compartmentalised 163 dendritic structures of neurons (Frick and Johnston, 2005; Losonczy et al., 2008; Makara et al., 2009). The 164 165 plasticity of excitability of dendritic structures greatly increases the complexity and non-linearity of neural information processing and storage (Remy et al., 2010; Branco and Häusser, 2010; Spruston et al., 2016). 166

In the context of artificial neural networks, IP is modeled as a mechanism that modifies the excitability of a neuron in order to achieve a specified output distribution for a given input distribution (Stemmler and Koch, 1999; Triesch, 2005). This is done by manipulating the parameters of a transfer function, which transforms the internal neural state to an output. A commonly used transfer function is the logistic function, defined in Eq. (3):

$$g_{a,b}(x) = (1 + \exp(-ax - b))^{-1}.$$
 (3)

The *a*, *b* are termed gain and bias of the function g(x) and *x* is the input which is transferred to the output space. By choosing an appropriate gain and bias, the input may be scaled and shifted in order to cause a response in the desired part of the sigmoid function g(x). The objective of IP is to adjust the gain and bias such that for a given set of inputs *X* the corresponding set of outputs g(X) approximates a predefined target distribution. Hence, IP is an autonomous adaptation of the sigmoid transfer function.

The particular IP learning rule for adapting the parameters of the transfer function is achieved by 177 minimising the Kullback-Leibler-divergence (KLD) (Kullback and Leibler, 1951), such that the output 178 distribution of a neuron is close to the target distribution. A common target distribution is the exponential 179 distribution, as it reflects aspects of homeostasis, i.e. maximizing the transmitted information (entropy), 180 while minimizing the positive mean output activity (metabolic costs) defined by the mean of the distribution. 181 For logistic functions, Eq. (3), and the exponential as a target distribution, the learning rules have been 182 derived in (Triesch, 2005). The procedure will only be sketched in the following. For neurons using the 183 tanh as transfer function, see (Schrauwen et al., 2008). 184

$$L_{\mathrm{KL}}(f_g \mid\mid f_{\mathrm{exp}}) = \mathbb{E}_x \left[L_{\mathrm{KL}} \left(f_g \left(g_{a,b}(x) \right) \mid\mid \frac{1}{\mu} \exp\left(\frac{-g_{a,b}(x)}{\mu}\right) \right) \right]$$
$$= \int f_g \left(g_{a,b}(x) \right) \log\left(\frac{f_g \left(g_{a,b}(x) \right)}{\frac{1}{\mu} \exp\left(\frac{-g_{a,b}(x)}{\mu}\right)} \right) \mathrm{d}x \tag{4}$$

$$f_g(g_{a,b}(x)) = \frac{f_x(x)}{\frac{\partial g_{a,b}(x)}{\partial x}}$$
(5)

185 In Eq. (4), a loss function L is defined as the KLD L_{KL} with the probability distribution function f_g 186 of the outputs of the logistic transfer function $g_{a,b}(x)$ with respect to the exponential target distribution

 f_{exp} . The output distribution of the logistic function $g_{a,b}(x)$ is parameterised by a, b, while the parameter 187 188 of the exponential distribution is the mean μ . The output distribution $f_q(g_{a,b}(x))$ is defined as the input distribution $f_x(x)$ remapped by the sigmoid g(x), as in Eq. (5). Minimizing the KLD is done by taking 189 190 the derivative with respect to (a, b) and performing gradient descent with a learning rate of η (the step 191 size). For online updating rules (stochastic gradient descent) the inputs x are assumed to be independent and identically distributed. Then the true gradient of the expectation value can be approximated for an 192 193 appropriate choice of the learning rate. This leads to the learning rules for adaptation of the parameters gain a and bias b with the learning rate η (Triesch, 2005): 194

$$\Delta b = \eta \left(1 - \left(2 + \frac{1}{\mu} \right) g_{a,b}(x) + \frac{1}{\mu} g_{a,b}(x)^2 \right), \tag{6}$$

$$\Delta a = \frac{\eta}{a} + x \Delta b. \tag{7}$$

Besides this online adaptation rule, a batch version of IP was derived in (Neumann and Steil, 2011). 195 The application of IP has repeatedly been reported to improve performances in reservoir computing – a 196 particular form of computing with transients in dynamical systems - (Steil, 2007a,b; Schrauwen et al., 197 2008; Wardermann and Steil, 2007) as well as increasing the robustness with respect to the parameter 198 initialization (Neumann and Steil, 2011). It has been noticed that in a network of neurons adapted by IP the 199 target distribution is also approximated on the network level (Steil, 2007a). There have been a number 200 of variations of IP learning with respect to the target distribution, for further information see (Verstraeten 201 202 et al., 2007; Schrauwen et al., 2008; Boedecker et al., 2009b,a). Furthermore, combinations of IP with other forms of plasticity have been investigated, e.g. with Hebbian learning, which leads to identification of 203 independent components in the input (Triesch, 2007). 204

Finally, it should be noted that IP leads to instability of recurrent neural networks (RNN). In Marković and Gros (2010, 2012) the authors show that introduction of IP in RNN leads to the destruction of the attractor stability, resulting in spontaneous and continuously ongoing activity for networks without and with very small input amplitudes. The result of RNN destabilisation by IP has also been confirmed in spiking neural networks (Lazar et al., 2007). These destabilizing effects on the dynamics are relevant for applying IP in dynamic neural fields as will be discussed in this paper.

211 The adaptation of the intrinsic plasticity via stochastic gradient descent can be optimized by utilizing the concept of a natural gradient, introduced in (Amari, 1998). It has been shown, that the metric structure of 212 the parametric space of neural networks has a Riemannian character (Amari, 1998). Thus the relationship 213 between the distance of two sets of parameters and the distance in the output space of the transfer function 214 defined by the loss function is non-linear. Adapting the conventional gradient with respect to the Riemannian 215 216 metric corrects for this non-linearity, such that the distance of two parameter sets linearly transfers to 217 the output space, measured by the loss function. This change of the gradient is termed *natural gradient* and leads to a substantial performance increase in the convergence rate for IP(Neumann and Steil, 2012; 218 Neumann et al., 2013). Therefore the natural gradient is used in this paper due to these technical benefits, 219 220 although the adaptation of DNFs with IP proposed in this paper in principal also works with the standard 221 IP adaptation.

A natural gradient-based parameter adaptation for IP termed NIP has been derived in (Neumann and Steil, 2012), here only the resulting learning rules are given:

$$\Delta \vec{\theta} = -\eta \left(F(\vec{\theta}) + \epsilon \mathbb{I} \right)^{-1} \nabla_E L_{\text{KL}} (f_g \mid\mid f_{\text{exp}}), \tag{8}$$
$$= -\eta \nabla_E L_{\text{KL}} (f_g \mid\mid f_{\text{exp}})$$

$$F(\vec{\theta}) = \mathbb{E}_{x} \Big[\nabla_{E} L_{\mathrm{KL}} \big(f_{g} \mid\mid f_{\mathrm{exp}} \big) \cdot \nabla_{E}^{T} L_{\mathrm{KL}} \big(f_{g} \mid\mid f_{\mathrm{exp}} \big) \Big].$$
(9)

The standard gradient of the loss function in an Euclidean metric ∇_E is transformed into a gradient in the Riemannian metric ∇_F by inverting the Matrix $F(\vec{\theta})$, which is the Fisher information, i.e. the Riemannian metric tensor. In order to prevent numerical instabilities of the tensor inversion, a Tikhonov regularization is applied by adding the identity matrix I with a small regularization factor ϵ to the tensor $F(\vec{\theta})$ before inversion. Just as before, the loss function L_{KL} is the KLD for neuron output $g_{\theta}(x)$ and parameters a, b. As the needed expectation value of the gradient with respect to the input in Eq. (9) is not available in an online framework, the tensor $F(\vec{\theta})$ is estimated online by:

$$\hat{F}_{t+1}(\vec{\theta}) = (1-\lambda)\hat{F}_t(\vec{\theta}) + \lambda \nabla_E L_{\mathrm{KL}}(f_g \parallel f_{\mathrm{exp}}) \cdot \nabla_E^T L_{\mathrm{KL}}(f_g \parallel f_{\mathrm{exp}}), \tag{10}$$

with λ realizing a low pass filter with exponential decay which is set to 0.01. For computational efficacy, the inversion of the tensor *F* in every time step (Eq. (8)) can be circumvented by directly estimating the inverse tensor F^{-1} as described in (Park et al., 2000). Using NIP gives a good approximation of the gradient direction in parameter space, which is also confirmed by the experiments in the evaluation section.

3 INTRINSIC PLASTICITY FOR DYNAMIC NEURAL FIELDS

Intrinsic plasticity (IP) is a local adaptative mechanism that models the autonomous adaptation of the 235 sensitivity (gain) and threshold (bias) of a single neuron in order to match the statistics of the neuron's 236 output to a predefined target distribution. We apply this idea to DNFs with respect to a global gain and 237 a global bias parameter that control the entire population of neurons in a DNF. DNFs are a mean field 238 approximations of a heterogeneous recurrent networks to capture the qualitative, global patterns. Our 239 proposed application of IP on a population level directly tunes the DNF output distribution and therefore 240 achieves the same effect (on the network level) as IP in single neuron would. Thus, conceptually IP in 241 DNFs captures the qualitative, global pattern change in a network as would result form IP in every single 242 neuron. 243

DNFs are consistent with population coding, in which the value of a feature is encoded by the activity of 244 those neurons within a population that are broadly tuned to that value. If particular feature values never 245 occur in the input, the corresponding neurons never become active. If the gain and bias of each neuron 246 would be adapted individually, this would lead each neuron to approximate the desired target distribution. 247 The output of the population would converge to an uniform distribution of feature values, reducing the 248 information encoded in the population. In contrast, the adaptation of a *global* gain and bias for all neurons 249 in a population of a DNF proposed here ensures that the encoding of the input in the DNF activity remains 250 stable. In the Discussion we briefly review evidence from computational neuroscience that supports this 251 notion of global adaptation. 252

To implement IP in a DNF, the field equation needs to be slightly reformulated. The standard formula of a DNF is given in Eq. (1), where the logistic transfer function $g(\cdot)$ is now used in the parametric version:

$$g_{a,b}(x) = \left(1 + \exp\left(-ax + b\right)\right)^{-1} \tag{11}$$

The gain, *a*, controls the steepness of the sigmoid and the bias parameter, *b*, controls the position of the zero-crossing of the sigmoid. The bias defines a gain dependent resting level, b = ah, which replaces the former static resting level, *h*, in Eq. (1). The gain, *a*, scales all weights, i.e. is a scaling factor for the input S(x, t) together with the recurrent, lateral interaction kernel, $\omega(x, x')$, in Eq. (1). As all the weights are jointly scaled, the relative contributions of input signal and lateral interaction remain fixed for a static input.

260 Furthermore, three design choices have to be made for deriving the IP learning rules:

- 261 1. Define a scalar measure, z, of the input of the field.
- 262 2. Define a scalar measure, y, of the output of the field.
- 263 3. Chose the desired target output distribution.

Regarding the first two points, the *output measure*, y(t), of the field is defined as the maximum output of the neural field:

$$y(t) = \max_{x} \left(g_{a,b} \left(u(x,t) \right) \right). \tag{12}$$

266 The *input measure*, z(t), of the field is given by the field activation at the position of the maximum output:

$$z(t) = u \left(\operatorname*{argmax}_{x} \left[g_{a,b} \left(u(x,t) \right) \right], t \right).$$
(13)

Hence, the input for IP is a composition of the actual field input and lateral field interactions, reflecting recurrent components of the neural dynamics. The main advantage of this measure is that it does not alter the output range. If the field output activity is in the range of (0, 1), for instance, the max (\cdot) is in that range too. This removes the need for an additional processing step of input normalization and parameter tuning.

Two alternative definitions would be the integrated (i.e. summed) or the mean of the field output activity. In contrast to the maximum, these are sensitive to the particular parametrisation of the recurrent lateral interaction kernel (i.e. the peak size) with respect to the DNF size. Hence, both of these alternative measures require a tuning of the target distribution parameters with respect to the particular DNF parametrization and are therefore neglected. Moreover, choosing the integrated output activity of a DNF as field output would make the output distribution more sensitive to the simultaneous occurrence of multiple peaks.

The *target output distribution* of y is set to the exponential distribution with mean μ , implying a sparseness constraint on the field output with respect to the output over time:

$$T(y(t)) = \frac{1}{\mu} e^{-\frac{y(t)}{\mu}}.$$
(14)

The exponential distribution is particularly suited when the DNF output is desired to be near zero for the majority of inputs (i.e. most of the time) and output activity is only required for a minority of the inputs. Furthermore the exponential is the maximum entropy probability distribution for a specified mean which is optimal with respect to the information transfer. Thus, an exponentially distributed DNF output corresponds



to an optimization of the information encoding in the DNF which remains stable during changes in the input statistics, e.g. mean or variance.

With these design choices, the optimization problem is equivalent to the one in (Triesch, 2005) (described in Sec. 2.2) and the learning rules for adapting the gain a and bias b are given by:

$$\frac{\Delta a}{\Delta t} = \frac{\eta}{a} + \frac{\Delta b}{\Delta t} z(t), \qquad (15)$$

$$\frac{\Delta b}{\Delta t} = \eta \left(1 - \left(2 + \frac{1}{\mu} \right) y(t) + \frac{1}{\mu} y(t)^2 \right).$$
(16)

287 The learning rate η is set to 0.001 and μ to 0.2.

Concerning the impact of IP on the stability of the DNF dynamics, it should be noted that IP drives the 288 dynamics towards the detection instability, i.e. to the "edge of stability". This becomes apparent, when 289 inspecting the behavior of the learning equations Eq. (15) and Eq. (16), depicted in Fig. 3. It is visible, 290 that for high DNF output the bias (i.e. resting level) is decreased while for low DNF output the bias is 291 increased, independent of the input. While the gain adaptation depends on the bias adaptation and the 292 current input, the principal "direction" of adaptation is the same as for the bias: high output with high input 293 leads to a decrease of the gain, low output with high input leads to the gain increase. Hence, the parameters 294 are adapted such that IP leads to the destruction of the attractor in which the system state currently is. In 295 the long run, this enforces an oscillation between the two attractors (the stable fixed points a) and c) in 296 Fig. 2), i.e. drives the system repeatedly through the detection instability. Therefore, IP prevents the DNF 297 298 from operating in a self-stabilizing regime where the recurrent interaction is sufficient for maintaining the system output, independent of the input. A DNF with IP will operate in a regime where the system state 299 regularly runs through the detection instability – driven by the input. 300

The parameter adaption of IP can be significantly improved with respect to the convergence speed and robustness by computing the natural gradient (Sec. 2.2). Therefore, the gradient direction and amplitude of Δa and Δb is corrected by a metric tensor imposing a Riemannian structure in parameter space, i.e. the natural gradient is computed as described in (Neumann et al., 2013). The tensor decay parameter λ in Eq. (10) is set to $\frac{\tau}{1000}$ where τ is the time constant of the DNF equation in Eq. (1). The regularization parameter ϵ in Eq. (8) of the tensor inversion is set to 0.0001.

4 EVALUATION

For evaluating the DNF with IP, an input time series is constructed from haptic recordings of robotic object 307 manipulations conducted in Strub et al. (2014a). Two features were used from these data, the orientation of 308 a contact ([0, 360 deg]) and its spatial shape on the two-dimensional tactile sensor arrays. The spatial shape 309 310 of the contact is rated between (0, 1), dependent on how "circular" its shape is: 1 corresponds to a perfect circular contact shape and 0 corresponds to a sharp line on the tactile sensor. As the object manipulations 311 312 are done with two fingers, there may be none, one, or two simultaneous contacts at every point in time (originating from both fingers but with opposing contact orientations). A one dimensional population code 313 is generated from these two features, as depicted in Fig. 4. A population of neurons encodes the contact 314 315 shape over the contact orientation, where the output rate of each neuron (bars in Fig. 4) signals the presence of a circular contact at the orientation which the neuron encodes (position along the x axis in Fig. 4). The 316 neuron response is blurred with a Gaussian filter across the contact orientation dimension, depicted by 317 318 the blue bars in Fig. 4. This population encoding of the tactile input is accordingly done for every step in time and the resulting time series of the recorded dataset is looped in order to present it for arbitrarily long 319 320 periods.

321 With this setup, the following cases are evaluated (an average one and three limit cases):

- 322 1. input with average amplitudes $S(x) \in [0, 6]$ for $\mu = 0.1$ and $\mu = 0.2$;
- 323 2. input with low amplitudes $S(x) \in [0, 1]$ (i.e. $\div 6$);
- 324 3. input with high amplitudes $S(x) \in [0, 36]$ (i.e. $\times 6$);
- 4. input with high offsets $S(x) \in [-12, -6]$ (i.e. -12) for IP with and without natural gradient.

These limit cases were selected, since they are quite common in situations when DNFs are driven with sensory inputs and lead to incorrect behavior: high amplitude input might saturate the field, whereas low amplitude might render the field unresponsive. Both effects can occur if input distribution is scaled or shifted. The goal in all these experiments is to detect the most circular contacts with the DNF, i.e. the DNF output should give a peak if the relative input "circularity" is sufficient to be classified as a circular contact and have zero output otherwise. This classification into two classes depends on the particular distribution of the circularity feature.

333 4.1 Varying the mean

In the first set of experiments, the input time series S(x, t) is fed into a one-dimensional DNF with IP for two different means ($\mu = 0.1$ and $\mu = 0.2$) of the target exponential distribution. These values are in the range of biological neurons in the cortex, see e.g. (Barth and Poulet, 2012; Hromádka et al., 2008; Margolis



Figure 4. Sketch of the input encoding used for evaluation of DNFs with IP, illustrated for two tactile contacts at opposing orientations ($x = 95^{\circ}$ and $x = 275^{\circ}$). A population of neurons encode the contact circularity over contact orientation, with each neuron encoding a specific orientation. The corresponding neurons representing the orientations of the tactile inputs are activated and their response strength is related to the contact circularity of the tactile contacts (the two black bars). The Gaussian blurring of the neuronal activation to neighboring neurons (encoding similar orientations) is depicted in the blue bars. This population representation of tactile inputs is done for every time step, leading to the input time series S(x, t).



Figure 5. Selection of the input time sequence S(x, t) and the corresponding DNF output g(u(x, t)) for converged gain and bias adaptation. Time is on the x-axis and the one dimensional population code is on the y-axis. In the top plot the color encodes the input amplitude S(x) at the corresponding contact orientation x (y-axis) for a point in time (x-axis). In the middle and bottom plots the color encodes the DNF output activity g(u), i.e. surface detection at the corresponding contact orientation x (y-axis). Top: The input time series S(x) to the DNF. Middle ($\mu = 0.1$): the DNF output for the converged IP parameters (a = 0.65 and b = -3.5). The input-output correlation (Eq. (17)) for the shown sequence is 0.69. Bottom: The DNF output for the converged IP parameters (a = 0.59 and b = -3.0) for $\mu = 0.2$. The input-output correlation is 0.67.

et al., 2012). The aim here is to point out the qualitative influence of the target distribution mean on theDNF output.

The recurrent interaction kernel is parametrized with: $c_{exc} = 14$, $\sigma_{exc} = 2$, $c_{inh} = -7$, $\sigma_{inh} = 6$ and the DNF is sampled at 100 points (i.e. a size of [1,100]). The setup is run with presenting the input time series based on recorded data in realtime (3fps) and the DNF with IP has a τ of 100ms and is updated with an Euler step width of 10ms. The DNF with IP is run until the parameter adaptation by IP does not change qualitatively, i.e. it has converged.

A selection of the input sequence and the corresponding output sequence of the DNF in this setup is shown in Fig. 5. In the top row the DNF input amplitudes (i.e. intensity) S(x) is shown for contact orientations along the y-axis of the plot for a given point in time (x-axis). The dark regions encode high input amplitudes at the corresponding contact orientation (y-axis), see the colorbars on the right of the figure. The corresponding output of the DNF for the converged IP parameters is shown in the bottom row of Fig. 5.

It is noticeable that the processing by the DNF results in a "sharpened" version of the input, where the structure is preserved. The IP hyper-parameter μ determines how "sensitive" the DNF is with respect to the input: for $\mu = 0.1$ peaks are only generated for the highest input intensities, for a mean of $\mu = 0.2$ the DNF generates more peaks in time which also tend to last longer. The difference in the converged parameters between the two cases is a decrease of the gain *a* of 0.06 (-10%) and an increase of the bias *b* by 0.5 (+15%).

356 4.2 Varying the input distribution

In the following set of experiments the impact of a sudden change in the input distribution is analyzed. This could result e.g. from tactile exploration of a new object with different geometry (i.e. circularity distribution), changes in the tactile exploration speed or strategy. For this the input sequence is presented for four cycles (i.e. 20 minutes in the experimental setup) in order to let the IP parameter adaptation converge. After the 20th minute, the input amplitude S(x) is manipulated in its variance (scaled) or its mean (shifted). Then the parameter adaptation by IP is analyzed for the succeeding 30 minutes. The results of this evaluation are shown in the Figs. 6–9.

364 These figures show histograms of the unmodified input z(t) (A) as defined in Eq. (13) and the output of 365 the DNF y(t) for the original input (B) defined in Eq. (12). In (C) the DNF output y(t) histogram is shown 366 after learning has adapted the system to the manipulated input statistics and the experiment is stopped 367 (i.e. after 50 minutes). All histograms are computed within a 5min time window. Furthermore, in (D) 368 the output y(t) histogram (y axis) is plotted over time (x axis) with a 5 minutes sliding time window to 369 estimate the distribution. The size of the time window was chosen such that it contains one full input period 370 (approximately 5:15min with 3.33fps) representative for the input sequence. The color intensity encodes the occurrence of the output value during this time window, where white corresponds to no occurrence 371 372 and black to 100+ occurrences (similar to the plots A-C). For an enhanced visualization of the output 373 histogram over time in (D), it is additionally plotted on a logarithmic color scale in (E). The corresponding development of the gain over time is plotted in (F), and the bias in (G). The correlation of the maximum 374 375 DNF output y(t) with the corresponding input z(t) of the DNF is shown in (H), computed for a sliding 376 time window located at time step t:

$$corr(t) = \frac{\sum_{i=t-l}^{t} (z(i) - \bar{z})(y(i) - \bar{y})}{\sqrt{\sum_{i=t-l}^{t} (z(i) - \bar{z})^2 \sum_{i=t-l}^{t} (y(i) - \bar{y})^2}},$$

$$\bar{z} = \frac{1}{l} \sum_{i=t-l}^{t} z(i).$$
(17)
(18)

The mean of the output \bar{y} is computed analogous to \bar{z} (18). The length of the time window l is set to 5min, just as for the computation of the sliding output histogram plots mentioned above. In the following, the results of the evaluation for different manipulations of the input statistics are presented.

380 4.2.1 Low Amplitude

In the first of this set of experiments, the input is down-scaled in its amplitude from a range of [0, 6] to [0, 1]. The results are shown in Fig. 6.

The DNF is initialized with a bias (i.e. resting level) of -5 and a gain of 1 and has a recurrent interaction kernel which is kept constant for all experiments in this paper. The parameter adaption by IP results in the DNF output distribution shown in Fig. 6 (B) for the original input z(t). At the 20th minute the input is down-scaled by the factor of 6, which is too low to initiate DNF output activity. However, the gain and bias are adapted such that the DNF output distribution is restored within 10 minutes in the experiments. In particular mainly the gain is adapted, the bias remains in its regime, which is expected as the input variance is manipulated. In the plots (D) and (E) this drop and the recovery in the output activity y(t) is visible in the



Figure 6. DNF with IP for **low amplitude** input after the 20th minute. (A) DNF input z(t) histogram, (B) DNF output y(t) histogram after IP parameter convergence at the 20th minute. (C) DNF output y(t) histogram at the 50th minute after the input down-scaling. (D) DNF output histogram over time, (E) logarithmic version of (D), (F-H) gain, bias and the input-output correlation over time, respectively. See text for further description.

histogram "gap" around the 25th minute. The absence of DNF output is partly obscured, as the histogram
is computed within a 5 minute time window and the IP parameter adaption is completed within a similar
time frame as visible in (F) and (G).

While the output distribution is mostly restored by IP for a down-scaled input signal, the massive drop in 393 the input-output correlation in Fig. 6 (H) indicates an additional aspect of the adaption with IP. The input 394 signal S(x,t) is scaled by the gain a together with the recurrent interaction kernel $\omega(x-x')$. As the gain is 395 increased in order to compensate the decrease in the input signal intensity, the recurrent interaction is also 396 increased. Thus, in this case there is a shift in the relative contributions of input and recurrent "feedback" to 397 the current activity state u(x, t) of the DNF. The increased relative contribution of the recurrent component 398 increases the stabilization of the DNF output and thus, reduces the input output correlation. This effect is 399 also visible when comparing the final output distribution for the down-scaled signal in (C) with the output 400 distribution in (B) for the original input signal. In (C) there is an increase in the "high" output states near 401 one and a decrease in the "medium" output activity. 402

403 4.2.2 High Amplitude

The second experiment with respect to varying the input statistics is analogous to the previous, except that the input is now scaled-up. After the initial parameter convergence to the original input signal, the input is scaled to [0, 36] at the 20th minute. The re-adaptation of the IP parameters is then analyzed in Fig. 7.

After the up-scaling of the input S(x), the DNF output is driven into saturation for the majority of all inputs. This is reflected in the plots (D) and (F) in Fig. 7, where a change in the output distribution y(t) is visible. Like in the previous experiment, the effect is partly obscured by the temporal integration within a 5 minute time window in order to compute the histogram. As a consequence to the input up-scaling, the gain is lowered (F) for an appropriate re-scaling of the input signal. Similar as in the previous case of a lowered input amplitude, the bias remains stable. The final output distribution in (C) and at the 50th minute in (D) and (E) shows that the parameter adaptation by IP is capable to retain the desired target output distribution.



Figure 7. DNF with IP for **high amplitude** input after the 20th minute. (A) DNF input z(t) histogram, (B) DNF output y(t) histogram after IP parameter convergence at the 20th minute. (C) DNF output y(t) histogram at the 50th minute after the input up-scaling. (D) DNF output distribution over time, (E) logarithmic version of (D), (F-H) gain, bias and the input-output correlation over time, respectively. See text for further description.

415 However, just as in the previous experiment, the compensation of a re-scaled input signal S(x) with the 416 gain parameter shifts the relative contributions of input and recurrent interactions, in this case towards 417 a higher contribution of the input signal. As the gain parameter is decreased, the recurrent interactions 418 are weakened in their contribution to the DNF activation. Thus output peaks are less stabilized with 419 respect to input fluctuations. This is partially visible in the input-output correlation plot (H) in Fig. 7. Here 420 the correlation reaches one, implying a strongly input driven system output. This decrease of recurrent 421 interactions is also visible when comparing the output distributions before (B) and after (C) the input 422 up-scaling. While for the original input the IP parameters lead to a suppression of intermediate outputs, these are more prevalent after the input up-scaling. 423

424 4.2.3 High Offset

In the last experiment, the input signal S(x, t) is shifted in its mean by 12, thus from the range [0, 6]to [-12, -6]. In contrast to the previous two experiments, in which the input signal was scaled, this experiment requires an adequate adaptation of the bias only in order to compensate the input shift. This experiment is further utilized to illustrate the impact of the natural gradient in the gradient descent. For this, the experiment is carried out twice: first the IP adaptation with the natural gradient (NG) will be described as before and then the case of adapting IP without the NG is compared.

In the left column in Fig. 8 the case of the adaption with NG is illustrated, analogous to the previous experiments. After the 20th minute the input S(x) is shifted, which leads to decreased output activity in the output histograms (D1) and (E1). As visible in Fig. 9 (G), the bias is adapted such that it compensates the shifted input signal. Although the gain is initially modified, it converges back to the previous value, which will be discussed in a subsequent paragraph. When comparing the DNF output distributions in (B1) and (C1), no difference is noticeable. This also holds for the output histograms (D1 & E1), which look the same at the 50th minute as before the input shift at the 20th minute.

In contrast to the previous experiments, the gain is ultimately not adapted, such that the relative contributions from the input signal and recurrent interactions remain the same. The input manipulation can be fully compensated by the additive bias. Therefore the input-output correlation in (H) also converges back to the previous value.



Figure 8. DNF with IP and shifted input after the 20th minute with- and without the natural gradient. On the left three rows show the results for IP with natural gradient descent. The right three rows show the results when using the gradient descent in euclidean parameter space. Shown are the input z(t) (A) and output y(t) (B-C) histograms of the DNF. The output histograms over time (D) and (E) show the output distributions over time, computed by a sliding time window of 5 minutes. See text for further description.



Figure 9. Parameter adaptation in a DNF with IP and **shifted input** after the 20th minute with- and without the natural gradient. The parameter adaptation is shown for the gain (F) and bias (G), and the input-output correlation (z(t), y(t)) (H) is plotted. The experiment with NG is stopped after the 50th minute, the experiment without NG is run until minute 100. See text for further description.

442 The decrease of the gain for inputs with high bias (i.e. shifts) is an "input variance overestimation" problem of the IP algorithm (Neumann et al., 2013). The input variance, i.e. the deviation of the input 443 signal from zero, can be reduced by lowering the gain, thereby reducing the error of the output distribution 444 with respect to the target exponential distribution. However, this is only a short term solution as for an 445 increasing bias the optimal gain returns to the former value. The standard gradient descent of IP learning 446 therefore drastically lowers the gain in order to increase it again when the bias has been adapted such that 447 the input mean is compensated, visible in the orange graph in Fig. 9 (F). In this case the computation of the 448 natural gradient, i.e. the transformation of the gradient from the Euclidean space into the Riemannian space 449 prevents the reduction of the gain to nearly zero and only leads to a slight input variance overestimation, 450

visible in the de- and increase of the gain around the 25th minute, shown in the blue graph in (F). At 451 452 this point in time the bias reaches a regime in which the input leads DNF output and the gain starts to converge back to the previous value. Thus, in this experiment the impact of the change of the gradient 453 454 metric on the gradient direction is directly visible, as the adaptation of the gain in a "wrong" direction is 455 reduced, compared to the adaptation without NG in (F). Although the two learning algorithms have the same learning rate of $\eta = 0.001$, the learning with NG is much faster. This is in particular visible when 456 457 comparing the parameter adaptations in Fig. 9 (F) and (G), but also when looking at the output histograms 458 in Fig. 8 (D2) and (E2). Note, that in the output distribution plotted in Fig. 8 (C2) the parameter adaptation 459 by IP has not converged yet. Altogether, the use of the NG leads to a significantly faster convergence with less fluctuations in the parameter adaptation. 460

5 DISCUSSION

In this paper, the adaptation of dynamic neural fields by intrinsic plasticity is proposed, analogous to IP in 461 single neuron models. The core idea behind our approach is, first, to define scalar measures of the input and 462 output of the whole DNF. Here, the maximum output and the input at the corresponding location on the 463 feature dimension are chosen. Second, a target distribution of the DNF output measure is defined, which 464 determines the statistics of the output. Since we selected the maximum output as the output measure, the 465 target distribution in our case characterizes the distribution of "peak", i.e. detection, vs. "no peak", i.e. 466 non-detection, states. In this paper, the exponential distribution is chosen, analogous to the conventional IP 467 learning in single neurons. However, the proposed approach is not limited to the exponential distribution, 468 other target distributions as e.g. the Gaussian may be used. The choice of this target distribution for IP will 469 470 shape the overall dynamics of the DNF. If the DNF output should spend more time in the activated state 471 the Kumaraswamy's double bounded distribution parameterized with a + b = 1.0 could be an interesting candidate Kumaraswamy (1980). 472

473 These design choices enable to derive learning rules for IP, which adapt the bias (i.e. resting level) and 474 the gain in order to approximate the target distribution of the DNF's output. For an appropriate kernel parametrization, IP ensures a highly input sensitive operating regime for the DNF dynamics, defined by the 475 476 hyper-parameters of the target distribution. Therefore, only the DNF recurrent interaction kernel parameters 477 remain to be tuned manually. This autonomous adaptation of the DNF resting level and gain is in particular relevant for architectures in which DNFs receive inputs with unknown distributions, but for which the 478 479 desired output distribution is known, as in our example in the introduction, where 20% of the most circular contacts should be detected as being "flat surfaces", i.e. should produce a suprathreshold activity peak. 480 Furthermore, a DNF with IP is capable to compensate moderate changes in the input amplitude (i.e. 481 482 variance) and mean - however, at the cost of a shift in the relative contributions of input and recurrent 483 interactions to the DNF output. This shift in relative contributions is a clear limitation of the proposed approach when large changes in the variance of the input signal is expected, as revealed in the high- and 484 low-samplitude experiments. 485

Adaptation in our model changes a *global* gain and bias for the entire neuronal population, modelled by the DNF, in contrast to independent adaptation of a local gain and bias for every neuron in the population. Formulation of IP for population codes in a local, single neuron based fashion is not straight-forward: In population encoding, the activity of a neural field encodes the confidence that the input feature has the value, to which the underlying neurons are tuned, i.e. the neuron has a local receptive field in the input space of the DNF. It is only for a neuron that encodes relevant feature values (i.e. the input regularly falls within the receptive field of the neuron) that an adaptation of intrinsic excitability makes sense. In order

to realize an individual adaptation of gains and biases of single neurons, there needs to be an additional 493 494 mechanism in place to adapt the receptive field position (i.e. the input weights) of each neuron, i.e. to tune the neuron to represent a new feature value. This would correspond to an adaptive feature resolution on the 495 496 population level, with fovea-like effects where more neurons / representational space is used for feature 497 value regions with high probability. The originally proposed algorithm of the self organizing maps (SOM) would be an example of such a receptive field tuning of a DNF (Kohonen, 1982). The problem here is the 498 strong dependence on stochastic, uncorrelated input required for training and maintaining the SOM, which 499 renders the SOM algorithm inapplicable for highly correlated in time inputs. This motivates the tuning of 500 global parameters for the entire population. 501

There is also a motivation from the biological perspective for the global adaptation based on IP. In 502 addition to the plasticity of excitability in individual neurons and their dendritic structures, accumulating 503 evidence exists of neuronal mechanisms that perform a multiplicative normalization of entire populations 504 of neurons (Carandini and Heeger, 2012). The existence of global, network-wide activity regulation in 505 addition to single neuron and synaptic adaptations is also proposed in Slomowitz et al. (2015) based on 506 recordings of cultured hippocampal networks. In particular, a coupled gain and bias adaptation among 507 neuronal populations has been proposed as an explanation for results from large-scale recordings in the 508 primary visual cortex (V1) (Lin et al., 2015). These biological findings additionally motivate the proposed 509 510 implementation of IP in DNFs in this paper, based on a coupled gain and bias for an entire population of neurons. 511

This paper also shows the limits of the adaptation by IP, in particular when the amplitude of the input signal (i.e. variance) is subject to strong changes. If the input amplitude declines too much, the increasing gain will eventually reach a regime, where the recurrent feedback self-stabilizes the DNF output – independent of the input. In this case the adaptation will lower the gain and bias again, leading to an on-off oscillation of the DNF output. This corresponds to the results by Marković and Gros (2010, 2012), where the authors demonstrate that IP leads to the destruction of the attractor stability, leading to oscillatory and bursting behavior of recurrent neural networks with no- or very small inputs.

Despite these limitations for strong changes in the input distribution, this paper shows that the adaptation 519 of DNFs with IP is feasible and can be used in applications, in which a DNF architecture is driven by 520 sensory inputs whose statistics is not known in advance or may change over time. Examples of such 521 applications could be, e.g., color vision at varying illumination, or auditory perception with different levels 522 of background noise. The benefit of this adaptation is that it simplifies tuning and allows application of 523 DNFs to inputs whose distribution is only roughly known (e.g. in terms of the min and max values) while the 524 525 desired distribution of DNF output can be specified in advance. In such cases, the definition of a recurrent interaction kernel and a desired output distribution with its hyper-parameter(s) drive self-adaptation of the 526 DNF^1 . 527

DISCLOSURE/CONFLICT-OF-INTEREST STATEMENT

528 The authors declare that the research was conducted in the absence of any commercial or financial 529 relationships that could be construed as a potential conflict of interest.

¹ Work presented here was accomplished using a software framework for development of dynamic neural fields architectures, cedar: cedar.ini.rub.de. Code and data can be made available on request.

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